Chaster 2 Summary

Definition 5: Odds

The odds in favour of an event A is

$$\boxed{\frac{P(A)}{1-P(A)}}$$

The odds against the event A is

$$\boxed{\frac{1-P(A)}{P(A)}}$$

Chapter 3 Summary

Properties of $\binom{n}{k}$

1. $n^{(k)} = \frac{n!}{(n-k)!} = n(n-1)^{k-1}$ for $k \geq 1$.

2. $\binom{n}{k} = \frac{n!}{k!(n-k)!} = \frac{n^{(k)}}{k!}$

3. $\binom{n}{k} = \binom{n}{n-k}$ for all $k = 0, 1, \dots, n$.

4. $\binom{n}{0} = \binom{n}{n} = 1$

5. $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$

6. Binomial Theorem: $(1+x)^n = \sum_{k=0}^{n} \binom{n}{k} x^k$

3.5 Notes

1. $n^{(k)} = n(n-1)\cdots(n-k+1)$

2. $n^{(0)} = \frac{n!}{(n-0)!} = 1$

   $0! = 1.$

3. $\binom{n}{k} = \frac{n^{(k)}}{k!} = \frac{n(n-1)\cdots(n-k+1)}{k!}$

4. When n and k are non-negative integers and $k > n$ then $\binom{n}{k} = \frac{n^{(k)}}{k!} = \frac{n(n-1)\cdots(1)(0)\cdots(n-k+1)}{k!} = 0$

Geometric Series

$$\boxed{\sum_{i=0}^{n-1} t^i = 1+t+t^2+\cdots+t^{n-1} = \frac{1-t^n}{1-t} \quad \text{for } t \neq 1}$$

$$\boxed{\sum_{x=0}^{\infty} t^x = 1+t+t^2+\cdots = \frac{1}{1-t} \quad \text{for } |t| \neq 1.}$$

Binomial Theorem

$$\boxed{(1+t)^n = \sum_{x=0}^{\infty} \binom{n}{x} t^x \quad \text{if } |t|<1}$$

Multinomial Theorem

A generalization of the Binomial Theorem is:

$$\boxed{(t_1+t_2+\cdots+t_k)^n = \sum \frac{n!}{x_1!x_2!\cdots x_k!} t_1^{x_1} t_2^{x_2}\cdots t_k^{x_k}}$$

Hypergeometric Identity

$$\boxed{\sum_{x=0}^{\infty} \binom{a}{x}\binom{b}{n-x} = \binom{a+b}{n}}$$

Exponential Series

$$\boxed{e^t = 1+t+\frac{t^2}{2!}+\frac{t^3}{3!} = \sum_{n=0}^{\infty} \frac{t^n}{n!} \quad \text{for all } t \in \mathbb{R}.}$$

$$\boxed{e^t = \lim_{n\to\infty}\left(1+\frac{t}{n}\right)^n \quad \text{for all } t \in \mathbb{R}}$$

Chapter 4 Summary

De Morgan's Laws

$$\boxed{\begin{aligned}\overline{A\cup B} &= \bar{A}\cap\bar{B}\\ \overline{A\cap B} &= \bar{A}\cup\bar{B}\end{aligned}}$$

Rule 4a: Addition Law of Probability or The Sum Rule

$$\boxed{P(A\cup B) = P(A) + P(B) - P(A\cap B)}$$

Rule 4b: Probability of the union of 3 events

$$\boxed{P(A\cup B\cup C) = P(A) + P(B) + P(C) - P(AB) - P(AC) - P(BC) + P(ABC)}$$

Definition 6: Mutually Exclusive

Events A and B are mutually exclusive if $A\cap B = \emptyset$ and $P(A\cap B) = P(\emptyset) = 0$.

Rule 5a:

Let A and B be mutually exclusive events. Then

$$\boxed{P(A\cup B) = P(A) + P(B)}$$

Rule 5b

Let $A_1, A_2, \ldots, A_n$ be mutually exclusive events. Then

$$\boxed{P(A_1\cup A_2\cup\cdots\cup A_n) = \sum_{i=1}^{n} P(A_i)}$$

Rule 6:

$$\boxed{P(A) = 1 - P(\bar{A})}$$

Definition 7: Independent Events

Events A and B are independent events if and only if

$$\boxed{P(A\cap B) = P(A)P(B)}$$

Definition 8: Independent Events

Events $A_1, A_2, \ldots, A_n$ are independent events if and only if

$$\boxed{P(A_{i_1}\cap A_{i_2}\cap\cdots\cap A_{i_k}) = P(A_{i_1})P(A_{i_2})\cdots P(A_{i_k})}$$

for all sets $(i_1, i_2, \ldots, i_k)$ of distinct subscripts chosen from $(1, 2, \ldots, n)$

Definition 9: Conditional Probability

The conditional probability of event A, given event B, is

$$\boxed{P(A|B) = \frac{P(A\cap B)}{P(B)}, \text{ provided } P(B) > 0.}$$

Rule 7: Product Rules

$$\boxed{\begin{aligned} &P(AB) = P(A)P(B|A) \\ &P(ABC) = P(A)P(B|A)P(C|AB) \\ &P(ABCD) = P(A)P(B|A)P(C|AB)P(D|ABC) \end{aligned}}$$

Rule 8: Law of Total Probability

Suppose that

$$\boxed{A_1 \cup A_2 \cup \cdots \cup A_k = S \text{ and } A_i \cap A_j = \emptyset \text{ if } i \neq j.}$$

Let B be an arbitrary event in S, then

$$\boxed{\begin{aligned} P(B) &= P(BA_1) + P(BA_2) + \cdots + P(BA_k) \\ &= \sum_{i=1}^{k} P(A_i)P(B|A_i) \end{aligned}}$$

Chapter 5 Summary

Hypergeometric Distribution

- A collection of N objects with 2 distinct types: success (S) and failure (F)
- There are r successes and N−r failures.
- Pick n objects without replacement.
- Let X be the number of successes obtained. Then
- X has a Hypergeometric distribution, with parameters N, r, n
- PF:

$$\boxed{f(x) = P(X=x) = \frac{\binom{r}{x}\binom{N-r}{n-x}}{\binom{N}{n}}}$$

Note: $x \leq \min(r, n)$

Binomial Distribution

- An "experiment" with 2 distinct outcomes: success (S) and failure (F)
- $P(S) = p$ and $P(F) = 1-p$
- Repeat the experiment n
- Let X be the number of successes obtained. Then,
- $X \sim \text{Binomial}(n, p)$
- PF:

$$\boxed{f(x) = P(X=x) = \binom{n}{x} p^x (1-p)^{n-x}}$$

Negative Binomial Distribution

- Two distinct types of outcome: S and F
- $P(S) = p$ on each trial (independent)
- Continue doing the experiment until k successes have occurred
- Let X be the number of failures before the 1st success. Then,
- $X \sim \text{Negative Binomial}(k, p)$
- PF:

$$\boxed{f(x) = P(X=x) = \binom{x+k-1}{x} p^k (1-p)^x} \quad \text{for } x = 0, 1, \ldots,\ 0 < p < 1$$

Geometric Distribution

- Two distinct outcome: S and F
- $P(S) = p$ on each trial (independent)
- Let X be the number of failures before the kth success. Then,
- $X \sim \text{Geometric}(p)$
- PF:

$$\boxed{f(x) = P(X=x) = (1-p)^x p} \quad \text{for } x = 0, 1, \ldots \text{ and } 0 < p < 1$$

Poisson Distribution from Binomial

- Approximate Binomial Distribution when n is large and p is small.
- PF:

$$\boxed{f(x) = P(X=x) = \frac{u^x e^{-u}}{x!} \,,} \qquad \text{for } x = 0, 1, \ldots$$

Poisson Distribution from Poisson Process

- Independence.
- Individuality: $P(2 \text{ or more events in } (t, t+\Delta t)) = o(\Delta t)$ as $\Delta t \to 0$.
- Homogenity or Uniformity: $P(\text{one event in } (t, t+\Delta t)) = \lambda \Delta t + o(\Delta t)$
- PF:

$$\boxed{f(x) = P(X=x) = \frac{(\lambda t)^x e^{-\lambda t}}{x!} \,,} \qquad \text{for } x = 0, 1, \ldots$$

Chapter 7 Summary

Definition 16 : Expected Value

Let X be a discrete random variable with range$(X) = A$ and probability function $f(x)$.
The expected value (also called the mean or the expectation) of X is

$$\boxed{E(X) = \sum_{x \in A} x f(x)}$$

Theorem 17 :

Let X be a discrete random variable with range$(X) = A$ and probability function $f(x)$
The expected value of some function $g(X)$ of X is given by

$$\boxed{E(g(X)) = \sum_{x \in A} g(x) f(x)}$$

Linearity of Expectation

For $a, b \in \mathbb{R}$ and functions g_1 and g_2

$$\boxed{E(ag_1(X) + bg_2(X)) = aE(g_1(X)) + bE(g_2(X))}$$

Expected Value of Poisson Distribution

If $X \sim \text{Poisson}(\mu)$, then

$$\boxed{E(X) = \mu.}$$

Expected Value of Binomial Distribution

If $X \sim \text{Binomial}(n, p)$, then

$$\boxed{E(X) = np.}$$

Expected Value of Geometric Distribution

If $X \sim \text{Geometric}(p)$, then

$$\boxed{E(X) = \frac{1-p}{p}}$$